PoTW 21: Week of 11-26-2021

Problem of the Week at shsmathteam.com

Submission form: link to submit

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Alternatively, you can message Andrew Liu on Facebook Messenger. Please don't be afraid to reach out for help, asking for hints is heavily encouraged if you feel stuck.

Problem of the Week #21: Contrived Turkey

Algebra kind of

To prepare Thanksgiving for his girlfriend, Jay buys infinitely many turkeys. Suppose that each turkey can be represented by a uniform cube of side length 1. Jay labels the turkeys from 2 to ∞, and uses the following scheme to cut the turkey labeled i:

- line up the turkey so that from birds-eye perspective it fills up the square on the cartesian plane with vertices at $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$.
- make two continuously curvy cuts, one along the line $y = x$, and one along the line $y = \sqrt[i]{1-x^i}$, where i is the label of the turkey he is cutting.
- his cutting splits the turkey into four regions. he gives himself two of these regions; the region with center of mass closest to the x-axis, and the region with center of mass farthest away from the x-axis.

Assume that he follows this cutting process for all $i \geq 2$, and let A_i be the volume of the turkey that he gives to himself for each i. Compute

$$\sum_{k=2}^{\infty} \frac{(-A_k)^k}{k}.$$