PoTW 29: Week of 2-18-2022

Problem of the Week at shsmathteam.com

Submission form: link to submit

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Alternatively, you can message Andrew Liu on Facebook Messenger. Please don't be afraid to reach out for help, asking for hints is heavily encouraged if you feel stuck.

Problem of the Week #29: Locus Practice

Geometry

(1) Consider a unit square S. A, B, C are vertices of a right triangle that has right angle at A, and points B, C lie in S. Find the area of the region of possible locations of A.

(2) Let $\mathcal{L}$ be the locus of points in the coordinate plane that consists of centers of circles that pass through the point $(0, 14)$ and are tangent to the x-axis. Compute the equation for $\mathcal{L}$.

(3) Circle Ω_1 has equation $x^2 + y^2 = 100$ and circle Ω_2 has equation $(x-6)^2 + y^2 = 4$. Let $\mathcal{L}$ be the locus of points in the coordinate plane that are the centers of circles externally tangent to Ω_2 and internally tangent to Ω_1. Compute the equation for $\mathcal{L}$.